Vernier Exhaust Perturbations on Radar and Altimeter Systems during a Lunar Landing

PAUL MOLMUD*
Space Technology Laboratories Inc., Redondo Beach, Calif.

The methods of predicting rocket exhaust interference on electromagnetic wave propagation are discussed. These methods are applied to the question of the interference due to the vernier exhaust on the Doppler and altimeter systems of a lunar landing vehicle. It is found that the exhaust does not significantly perturb any of these systems.

Introduction

A PROCEDURE that has been proposed to accomplish a soft lunar landing "is to use a high thrust main engine stage to remove most of the approach velocity in an open loop fashion, and a low thrust vernier engine stage to control the spacecraft to a soft landing. The vernier stage employs terminal sensors to measure altitude [altimeter] and velocity [Doppler], throttling to control descent rate, and may employ body maneuvering to control horizontal velocity."[1]

It is the purpose of this paper to make an estimate of the interference that the vernier rocket exhaust may exert on the performance of the Doppler and altimeter systems of the lunar landing vehicle.

The subject of rocket exhaust interference on missile guidance and communication has been under considerable study in the past. The most detailed studies on this topic are in the form of unpublished company reports and studies. We shall make use of the results and conclusions of some of these studies[2-4] to provide upper bound estimates of flame effects on the guidance of the lunar vehicle. The main results of these studies are as follows:

1) Molecular absorption and polarizability are insignificant in determining the electrical properties of the rocket exhaust; the free electrons play the dominant role.

2) The electron collision frequency is determined (and therefore predictable), in the main, by the presence of those molecules with permanent electric dipole moments.

3) The observed degree of ionization of liquid rocket exhausts at exit plane conditions is enormously larger than predicted by equilibrium theory. The ionization can be explained as due to the thermal ionization of low-ionization fuel impurities (e.g., sodium) in the combustion chamber followed by slow electron-ion recombination as the combustion products accelerate, expand, and cool through the thrust chamber. The possibility of chemi-ionization as the cause of the observed ionization is not ruled out, however.

4) Computer programs were developed to predict the spatial distribution of the exhaust gases (flow pattern) for vacuum operation of the rocket motors.

5) The exhaust gas flow patterns combined with knowledge of electron collision frequency and fractional ionization (considered frozen beyond the exit plane) allow prediction of the complex electrical conductivity throughout the exhaust flare. The propagation of electromagnetic waves (of not too large a wavelength) may then be predicted through this extended medium.

We apply these results to make upper-bound estimates of flame effects on the altimeter and Doppler systems of the lunar landing vehicle. Our procedure will be as follows:

1) To choose a suitable flow pattern for the vernier exhaust gases expanding into a vacuum. This will give us the gas density and temperature encountered by rays (assuming ray optics valid) drawn from the antennas to ground.

2) To compute the electron collision frequency in the exhaust, using the predicted densities, temperatures, composition, and the collision cross sections available in the literature.

3) Lacking empirical information on the actual ionization of the decomposition products, to predict an upper bound on the fractional ionization of the exhaust by computing the thermal ionization, at chamber temperatures, of the impurities reported in the propellant.

4) Combining 1–3, we predict the varying complex conductivity encountered by rays passing through the exhaust and by the Eikonal solution of the wave equation, and we determine their phase shifts and attenuations. The minimum radii of curvature of the rays will also be determined.

Method of Prediction

A. Flow Pattern

A flow pattern was chosen for a gas exiting into a vacuum with Mach number at exit plane of 4.93 and $\gamma = 1.30$. The flow was computed from the program of Petersen and Wang[3] and obtained from D. Perkins of Space Technology Laboratories (see Fig. 1).

B. Ray Tracing

Each Doppler antenna (13,300 mc) is located $28\frac{1}{2}$ in. from the nearest vernier motor and 37 in. from the next. The axes

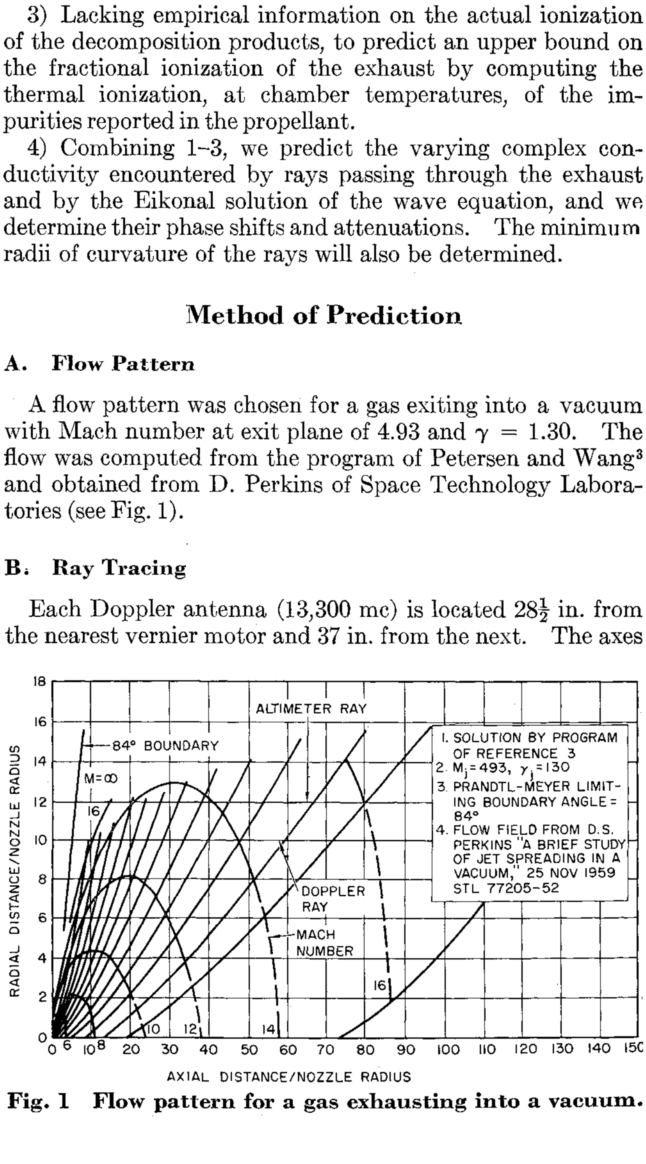

Fig. 1 Flow pattern for a gas exhausting into a vacuum.

Presented at the ARS Ions in Flames and Rocket Exhausts Conference, Palm Springs, Calif., October 10–12, 1962; revision received October 15, 1963. This paper was originally part of the preliminary design study submitted by Space Technology Laboratories to Jet Propulsion Laboratory under Jet Propulsion Laboratory Subcontract N3009. The author wishes to thank L. J. Van Nice of Space Technology Laboratories for supplying Refs. 20 and 21.

* Member, Senior Staff.

of the antennas make an angle of 20° with respect to the platform vertical, and the verniers are at zero angle. The altimeter (9200 mc) is $35\frac{1}{2}$ in. from the nearest vernier and 48 in. from the next motor (see Fig. 2).

We assume that the propagation of the forementioned frequencies through the vernier exhaust may be described in terms of ray optics (the Eikonal approximation) and, furthermore, that the rays propagate in very nearly straight lines. These assumptions will be justified subsequently.

Accordingly, to represent these rays on the flow pattern, we draw two straight lines parallel to the axis of flow; one at $28\frac{1}{2}$ from the axis (the Doppler ray) and one at $35\frac{1}{2}$ in. from the axis (the altimeter ray). The ionized gases intercepted by these rays will cause attenuation, phase shift, and bending (see Fig. 1).

C. Electromagnetic Theory

The conditions for the validity of the Eikonal solution are that the index of the medium must not change appreciably in a distance equal to a wavelength of the radiation, and that the amplitude of the wave must not change appreciably in the same distance.[7] We shall see that these conditions are satisfied in our application.

The Eikonal solution of the wave equation may be written as follows:

$$E(s) = E_0 \exp \int_0^s [i\alpha(s') - \beta(s')]ds' \qquad (1)$$

where E is the amplitude of the field, α and β are the phase and attenuation factors, respectively, and the integration is along the ray path. It is assumed that the index of refraction is very close to unity.

In an ionized medium where the wave frequency is much greater than either the collision frequency or the plasma frequency, α and β have the following simple forms[2]:

$$\alpha(s) = \frac{\omega}{c}\left(1 - \frac{\omega_p{}^2(s)}{\omega^2}\right)^{1/2} \approx \frac{\omega}{c}\left[1 - \frac{1}{2}\frac{\omega_p{}^2(s)}{\omega^2}\right] \qquad (2a)$$

where $\omega_p{}^2(s) = 4\pi n(s)e^2/m = 4\pi^2 f_p{}^2$, and f_p is the plasma frequency and $n(s)$ is the electron particle density at point s of the ray path:

$$\beta(s) = \frac{1}{2c}\frac{\omega_p{}^2(s)\nu(s)}{\omega^2} \qquad (2b)$$

where $\nu(s)$ is the collision frequency between electrons and the constituents of the exhaust cloud at point (s), and c is the velocity of light.

The radius of curvature R may also be written as[8]

$$\frac{1}{R} = \frac{\nabla\alpha}{\alpha}\sin\gamma = \frac{1}{2}\frac{1}{\omega^2}\nabla\omega_p{}^2 \sin\gamma \qquad (2c)$$

where γ is the angle between the direction of the ray and the gradient of the index.

D. Collision Frequency

We have made no measurements on the exhaust of the vernier engine, and so we have to make predictions of the expected electron density and collision frequency.

Our computation of collision frequency is based on cross-sectional data of the various constituents of the exhaust and work done by this author in translating such data to effective collision frequencies.[9]

The main ingredients of the exhaust are N_2, NH_3, H_2O, and H_2. For N_2, we have[10] $Q = 2.48 \times 10^{-23}\, v$(cgs), where Q is the cross section for momentum transfer for electron impact on N_2, and v is the relative velocity between an electron and an N_2 molecule.

The collision frequency between electrons and N_2 is given by[9]

$$\nu_{N_2} = \tfrac{5}{3} \times 6.5 \times 10^{10} \times C_{N_2} \times (p/p_{atm})$$

where C_{N_2} is mole fraction of N_2, p is the local pressure of the exhaust and p_{atm} is atmospheric pressure. For NH_3,[2, 11] $Qv^2 = 3.7$, and[9]

$$\nu_{NH_3} = \tfrac{2}{3} \times C_{NH_3} \times (\rho/T^{1/2}) \times 10^{-6} \times 7.6$$

where ρ is exhaust particle density and T the temperature (in degrees Kelvin), whereas for H_2O,[11] $Qv^2 = 5.9$, and[9]

$$\nu_{H_2O} = \tfrac{2}{3} + C_{H_2O} \times (\rho/T^{1/2}) \times 10^{-6} \times 12$$

and for H_2,[12]

$$Q = 1.45 \times 10^{-23} v + 8.9 \times 10^{-16}$$

and[9]

$$\nu_{H_2} = C_{H_2} \times \rho \times [\tfrac{4}{3} \times 5.55 \times 10^{-10} T^{1/2} + \tfrac{5}{3} \times 0.660 \times 10^{-11} T]$$

Under conditions of $\omega \gg \nu_{total}$, all of the separate computed frequencies ν_{N_2}, ν_{NH_3}, ν_{H_2O}, and ν_{H_2} may be added together and the result, ν_{total}, may be used in (2b). However, we find that, in the region of maximum gas density which the rays intercept, ν_{total} is determined in the main by ν_{NH_3} and ν_{H_2O}.

E. Electron Density

The electron density is perhaps the most difficult of all quantities to predict. Temperatures at the exit plane are so low for most rocket engines that one would expect very little ionization on a basis of purely thermodynamic equilibrium. However, experience tells us otherwise. The electron density in the exhausts of large-scale motors is significant and sometimes troublesome. These electrons can be explained only on a nonequilibrium basis.

We have had a fair degree of success in explaining the observed ionization in terms of the low-ionization potential contaminants that are found in the propellants.[4, 13] These contaminants are ionized in the combustion chamber where pressures and temperatures are fairly high, and thermodynamic equilibrium is assumed to be obtained. The ionized gases then flow from the combustion chamber through the expansion section. The electron-ion recombination rates and electron attachment rates are not fast enough to maintain continuous equilibrium. This recombination lag accounts for the abnormally high electron density at the exit plane.

In our computation of electron density, we shall proceed in a manner that is felt produces a vast exaggeration of the ionization to be expected in the vernier exhaust gases. We shall take the contaminants reported in the fuel and assume them to be ionized under the conditions of the combustion chamber. Then we allow the ionized gases to flow through the expansion section but do not allow any recombinations

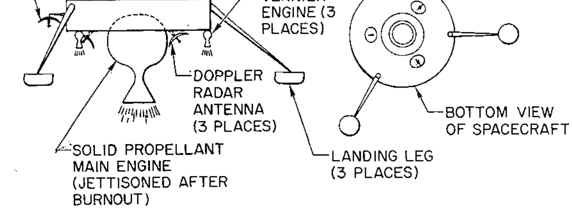

Fig. 2 Schematic representation of lunar soft landing spacecraft.[1]

Table 1 Composition of exhaust

Species	Mole fraction (C)
H_2O	0.120
N_2	0.293
NH_3	0.294
H_2	0.294

or any removal of electrons. Thus, the fractional ionization is to remain constant from the combustion chamber to infinite expansion.

Our computations of fractional ionization will be based on Saha's equation[14] applied to alkali metals:

$$\eta^2\rho/(C - \eta) = 2.38 \times 10^{15}\, T^{3/2} e^{-(I/T)\times 1.16\times 10^4} \quad (3)$$

where η is the fractional ionization of the exhaust (i.e., electron density divided by particle density), C the mole fraction of alkali atoms (i.e., potassium or sodium), I the ionization potential (in electron volts) of the alkali atom, and ρ the exhaust particle density at chamber conditions.

Pertinent Data and Computations

A. Engine Data

The propellant is hydrazine nitrate catalytically decomposed. The area ratio is $\epsilon = 50{:}1$, $P_c = 185$ psia, T_c 1644°K,† the maximum thrust = 220 lb, I_{sp} = 250 sec, the exit diameter = 5.8 in., and $\gamma = 1.30$. The expected residual water soluble impurities on analysis of the propellant is 0.00125% by weight.

The decomposition reaction is

$$0.25\ HNO_3 + 2.6\ N_2H_4 \rightarrow 0.75\ H_2O + 1.81\ N_2 + 1.83\ NH_3 + 1.83\ H_2$$

The computed engine data are $T_{exit} = 330$°K, $\rho_{exit} = 4 \times 10^{17}$ cm^{-3}, $M_{exit} = 5.07$, $\rho_{chamber} = 5.45 \times 10^{19}$ cm^{-3}. The composition of the exhaust is shown in Table 1.

B. Computation of Fractional Ionization of Exhaust

We have no chemical analysis available, but nonetheless we shall assume for our upper bound computation that the 0.00125% impurities reported is KCl. (A more reasonable assumption on the basis of relative abundance would be NaCl, but this would result in a predicted lower ionization.) These impurities are carried into the propellant via the reagent grade NH_4NO_3 employed in making the hydrazine nitrate. The mole fraction of K in the exhaust products is therefore 2.7×10^{-6}. Using Eq. (3), we determine the fractional ionization to be expected under combustion chamber conditions. The result is $\eta = 7 \times 10^{-10}$. We shall assume that recombination and attachment processes are negligible during the acceleration and expansion of the gases and, therefore, that the fractional ionization remains invariant up to infinite expansion. Our experience with large-scale engines indicates that it is more reasonable to expect a significant drop in fractional ionization as the gas flows from the combustion chamber to the exit plane. Our assumption of constant fractional ionization thus leads to a further exaggeration of electron densities.

It is instructive to compare the ionization we have computed for the vernier exhaust gases with the ionization observed in rocket motors burning organic nitrogen compounds with nitric acid but at much higher temperatures. We have no information as to the state of contamination of either the propellant or oxidizer used in these motors, but one might think that nitric acid would have at least the same trace concentrations of alkali metals as the reagent grade NH_4NO_3 employed in making the hydrazine nitrate used in the vernier engines.

Thus, Naval Research Laboratories[15] have made attenuation measurements on a motor burning monoethylaniline with nitric acid. The resultant measurements indicated a fractional ionization of close to 10^{-9}. Dauguet[16] burned a mixture of xylidine ($C_8H_{11}N$) and triethylamine [$N(C_2H_5)_3$] with HNO_3. The observed fractional ionization was 2×10^{-10}.

In view of the magnitude of these ionizations reported, but at higher flame temperatures, we feel justified in asserting that 7×10^{-10} is an upper bound on the fractional ionization to be observed in the vernier exhaust gases. Even if this were not so, we shall see subsequently that, because of the rarefied nature of the exhaust gases encountered by our Doppler and altimeter beams, ionization several orders of magnitude greater than the forementioned would be tolerable.

C. Computation of Collision Frequency

The Doppler and altimeter rays that we have drawn on the flow pattern of the vernier exhaust encounter gases that have a maximum density of the order of 10^{-3} of that of the gases at the exit plane and have a temperature of about 50°K (Mach 14). Using the relationships for collision frequency which we have previously described, we determine the collision frequency of electrons with each gas species under the foregoing conditions. These are $\nu_{N_2} = 9.1 \times 10^4$ sec^{-1}, $\nu_{NH_3} + \nu_{H_2O} = 1.38 \times 10^8$ sec^{-1}, and $\nu_{H_2} = 6.8 \times 10^5$ sec^{-1}. The sum of all these is $\nu_{total} = 1.38 \times 10^8$ sec^{-1}.

The collision frequencies under exit plane conditions are given by $\nu_{N_2} = 6.0 \times 10^8$ sec^{-1}, $\nu_{NH_3} + \nu_{H_2O} = 5.4 \times 10^{10}$ sec^{-1}, and $\nu_{H_2} = 2.0 \times 10^9$ sec^{-1}. Thus $\nu_{total} = 5.6 \times 10^{10}$ sec^{-1}.

Propagation Constants along Ray Path and Total Attenuation and Phase Shift

When the ray traverses a region where the gas density is 10^{-3} of that of the exit plane, then $\rho = 4 \times 10^{14}$ cm^{-3}. The electron densities and collision frequencies are given by $n = \rho\eta = 2.8 \times 10^5$ cm^{-3} and $\nu = 1.4 \times 10^8$ sec^{-1}. The propagation constants may now be computed according to (2a) and (2b). Thus, for a frequency of 10,000 mc,

$$\alpha = (\omega/c)[1 - 1.17 \times 10^{-7}]$$

$$\beta = 5.4 \times 10^{-10} \text{ nepers/cm}$$

Thus, the index of refraction is $1 - 1.17 \times 10^{-7}$, a negligible deviation from unity, and the attenuation per unit length is 4.7×10^{-4} db/km, truly insignificant.‡

The total phase shift and attenuation may be found by integrating $\alpha(s)$ and $\beta(s)$ along the ray path (from antenna to infinity) as in Eq. (1) (reading the densities off the flow pattern). We also include the radius of curvature of each ray. The results follow:

1) For the altimeter (9200 mc), the phase shift is 1.9×10^{-4} rad, the attenuation is 7.4×10^{-6} db, and the minimum

† Catalytic bed dimensions chosen for 40% ammonia decomposition. The flame temperature, T_c, was computed by Siegel for this degree of dissociation.[19] See also Refs. 20 and 21.

‡ The contributions to the index and absorption coefficient by NH_3 and H_2O, respectively, in this region and for a wave frequency of 13,300 mc are, for NH_3,[17]

$$\alpha = (\omega/c)[1 + 4 \times 10^{-9}]$$
$$\beta = 10^{-6} \text{ db/km}$$

and, for H_2O,[18]

$$\alpha = (\omega/c)[1 + 8 \times 10^{-12}]$$
$$\beta = 10^{-10} \text{ db/km}$$

The corresponding values of α and β for 9200 mc are considerably lower.

radius of curvature of ray $\sim 6 \times 10^{10}$ cm. [from Eq. (2c) and a scale length of about five exit radii].

2) For the Doppler (13,300), the phase shift is 2.7×10^{-4} rad, the attenuation is 1.2×10^{-5} db, and the minimum radius of curvature of ray $\sim 8 \times 10^{9}$ cm.

The magnitude of the computed radii of curvature provides full justification for the assumption of rectilinear propagation. The magnitudes of the phase shifts and attenuations provide ample leeway for any upward adjustment of our predicted ionization without causing any undue interference on the systems.§

The attenuation and phase shift of the Doppler ray were computed on a ray path parallel to the vernier axis of symmetry. Since the Doppler antenna is pointed 20° away from the axis of the vernier, the forementioned figures should be considered exaggerated.

Reflection Coefficient

One question that may be raised is whether much energy may be scattered off the exhaust back to the Doppler antennas. We seek to answer this by constructing a gross exaggeration of the actual conditions.

We assume that the Doppler wave is incident normally on an interface separating vacuum and the exhaust gases under their conditions at the exit plane. The magnitude of the reflected energy, which we shall determine, is taken as a gross measure of the efficacy of backscatter of the rocket exhaust. The reflected energy is larger for other than normal incidence; however, this energy is directed away from the antennas. This is why we consider only normal incidence.

It is fairly easy to show that the reflection coefficient for normal incidence for waves of frequency much higher than the plasma frequency may be written as

$$R = \frac{1}{16} \frac{\omega_p{}^4}{(\omega^2 + \nu^2)^2} \left[1 + \frac{\nu^2}{\omega^2}\right]$$

Then, for $f = 13{,}300$ Mc and $n = 2.8 \times 10^{8}$ cm^{-3}, $\nu = 5.6 \times 10^{10}$ sec^{-1} (exit conditions), and $R = 9.1 \times 10^{-10}$.

This size reflection coefficient corresponds to the reflected energy, being down by a factor of 90 db below the incident energy. We can expect even less energy reflected back from the actual rocket exhaust because the gases encountered by the Doppler beams are even more rarefied.

Conclusion

Gross exaggeration in estimating the ionization of the vernier exhaust has led to exceedingly small, in fact, almost nondetectable, effects on the Doppler and radar systems. Accordingly, we conclude that these systems will not be affected by the vernier exhaust at all.

§ If the potassium impurity were 100% ionized, then the foregoing phase shifts and attenuations would be increased by a factor of 4×10^{3}. However, this would require combustion chamber temperatures much higher than realizable under the stated conditions. Furthermore, even the factor of 4×10^{3} does not raise the perturbation to an intolerable level.

References

[1] Cheng, R. K. and Pfeffer, I., "A terminal guidance system for soft lunar landing," *ARS Progress in Astronautics and Rocketry: Guidance Control*, edited by R. E. Robertson and J. S. Farrior (Academic Press Inc., New York, 1962), Vol. 8, pp. 217–241.

[2] Altshuler, S., Moe, M., and Molmud, P., "The electromagnetics of the rocket exhaust," GM-TR-0165-00397, Space Technology Labs. (June 15, 1958).

[3] Peterson, J. B. and Wang, C. S., "Spreading of supersonic jets from axially symmetric nozzles," GM-TR-175, Space Technology Labs. (May 16, 1957).

[4] Molmud, P., "The old contaminant program and electron-ion recombination in expanding gases," Space Technology Labs. Rept. 9221.2-34 (March 1963).

[5] Calcote, H. F., "Ions in flames," Final Rept., Contract AF 04(647)-157, Aerochem. Res. Labs, Inc. (December 1960).

[6] Calcote, H. F., "Mechanisms for formation of ions in flames," Combust. Flame I, 385–403 (December 1957).

[7] Kerr, D. E., *Propagation of Short Radio Waves* (McGraw-Hill Book Co. Inc., New York, 1951), p. 54.

[8] Kerr, D. E., *Propagation of Short Radio Waves* (McGraw-Hill Book Co. Inc., New York, 1951), p. 44.

[9] Molmud, P., "Langevin equation and the a.c. conductivity of non-Maxwellian plasmas," Phys. Rev. **117**, 29–32 (April 1, 1959).

[10] Crompton, R. W., Huxley, L. G. H., and Sutton, D. J., "Experimental studies of the motion of slow electrons in air with application to the ionosphere," Proc. Roy. Soc. (London) **A218**, 507 (1953).

[11] Altshuler, S., "Theory of low-energy electron scattering by polar molecules," Phys. Rev. **107**, 114–117 (July 1, 1957).

[12] Bekefi, G. and Brown, S. C., "Collision cross section and energy loss of slow electrons in hydrogen," Phys. Rev. **112**, 159–164 (1958).

[13] Smith, F. T., "On the analysis of recombination reactions in an expanding gas stream," *Seventh Symposium (International) on Combustion* (Butterworths Scientific Publications, London, 1958), p. 93.

[14] Gaydon, A. and Wolfhardt, H. B., *Flames* (Macmillan Co., New York, 1960), pp. 302–305.

[15] Balwanz, W. W., private communication (1961).

[16] Dauguet, A., "Study of the ionization of the gases of propulsive jets," Ann. Telecommunications **12**, no. 6, 234 (1957).

[17] Townes, C. H. and Schawlow, A. L., *Microwave Spectroscopy* (McGraw-Hill Book Co. Inc., New York, 1955), p. 346, Figs 13 and 14.

[18] Kerr, D. E., *Propagation of Short Radio Waves* (McGraw-Hill Book Co. Inc., New York, 1951), pp. 660–661.

[19] Siegel, B., private communication, Space Technology Labs. (November 9, 1960).

[20] Grant, A. F., Jr., "Basic factors involved in the design and operation of catalytic monopropellant hydrazine reaction chambers," Jet Propulsion Lab. Rept. 20-77 (December 31, 1954).

[21] Lee, D. N., "Performance calculations for monopropellant hydrazine and monopropellant hydrazine-hydrazine nitrate mixtures," Jet Propulsion Lab. TR 32-348, p. 86, Fig. 7 (December 3, 1962).